

$$\dot{u} = \frac{1}{r} \frac{\partial \phi}{\partial z}, \quad \dot{\omega} = -\frac{1}{r} \frac{\partial \phi}{\partial r} \quad (24)$$

where ϕ is a velocity function, then the analogous differential equation for the time dependent flow of an arbitrary material becomes

$$\begin{aligned} & \frac{\bar{\sigma}}{\dot{\epsilon}} \nabla_1^4 \phi + \nabla_3^2 \phi \nabla_2^2 \left(\frac{\bar{\sigma}}{\dot{\epsilon}} \right) \\ & + \frac{2}{r} \frac{\partial}{\partial z} \left(\frac{\partial \phi}{\partial r} - \frac{2\phi}{r} \right) \frac{\partial}{\partial z} \left(\frac{\bar{\sigma}}{\dot{\epsilon}} \right) \\ & + 2 \frac{\partial}{\partial z} \left(2 \frac{\partial \phi}{\partial r} - \frac{\phi}{r} \right) \frac{\partial}{\partial r \partial z} \left(\frac{\bar{\sigma}}{\dot{\epsilon}} \right) = 0 \end{aligned} \quad (25)$$

In the case of a Newtonian fluid, the ratio of the shearing stress to the rate of shear strain is a constant; and this constant is usually called the coefficient of viscosity μ . Using analogous terms, the following relation can be written for a Newtonian fluid.